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Note

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Wake field in a magnetized dusty plasma with streaming ions

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Abstract : Three-dimensional wake potential for a stationary test dust particulate in uniformly magnetized dusty plasma with a continuous flow of ions, has been studied theoretically. Two standard directions of the external static magnetic field with respect to the direction of the flow of ions, have been considered. For magnetic field along the ion-flow direction, the lattice spacing along this direction is affected significantly for stronger magnetic field and unaffected for weaker magnetic field, while the spacing in the perpendicular plane is unaffected for both high- and low-density plasma. However, for magnetic field perpendicular to the ion-flow direction, the wake potential becomes weaker with large (constant) spacing along (perpendicular) the direction of the flow of ions in a high-density plasma.

Keywords : Wake potential, magnetized dusty plasma, streaming ions.

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A gaseous system of electrons, ions, neutral atoms/ molecules, and a dispersed phase of micron/sub-micron sized fine particles is usually referred to as a dusty plasma. Since the dust grain occur in plasma or radiative environments in laboratory and space conditions, they can accumulate electric charges on their surfaces due to a number of processes, such as, plasma currents, photoionization, secondary electron impact emission, and other competing processes. The occurrences of dust particles, physics of their charging and the consequences, their role in the process of coagulation and structure formation, such as, double layers in interstellar spaces, formation of stars and other heavenly bodies as well as the various possible applications with special reference to dust crystal experiments, waves and instabilities, nonlinear effects, *etc.* are presented in details in recent review articles [1–3].

Various communities, such as, space and astro-physicists, environ mentalists, industrial physicists, basic plasma researchers, *etc.* are all discovering and applying increasing new properties of dusty plasmas.

One of the great hopes of dusty plasma research lies in creating crystals of new materials in a process known as the dust Coulomb crystallization in the laboratory conditions [4]. A number of laboratory experiments [5–13], have clearly demonstrated the formation and other properties of macroscopic Coulomb crystals of solid dust-particles of micron size with lattice parameters of a few hundred microns.

It is now generally accepted that the formation of the dust Coulomb crystals can be explained in terms of attractive wake potential in a low temperature unmagnetized dusty plasma [14]. The oscillatory wake field is generated due to the interaction of the charged dust particulates and the collective modes of the plasma in the presence of streaming ions in the plasma [15–17]. The ion streaming is

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spontaneously established in the sheath region of the laboratory dust-crystal experiments because of the relatively faster thermal motion of electrons. In the uniformly moving ion-frame, the dust particulates can be assumed moving and their interaction with the ion-acoustic perturbation in the ion-frame produces what is known as the wake potential, which contributes to the formation of the periodic structure.

In recent studies, the formation of the wake potential is shown to be more effective due to the resonant interaction of the slowly moving grains and the extremely low-frequency (lf) and low-phase-velocity dust-acoustic waves in unmagnetized, cold, and collisionless dusty plasma [18,19]. However, these theories are valid for predicting one-dimensional structure along the motion of the dust particulates.

An external static magnetic field is usually applied to laboratory plasma experiments for better control and confinement, and other purposes [20,21]. A few attempts [22–25] have been made to understand the role of external magnetic field in dust crystal experiments. Recently, Nambu *et al* [26] (hereafter referred as Paper I) have clarified the role of external magnetic field on the periodic attractive forces in the dusty plasma. However, they confined their studies in one dimension along the direction of streaming ions. In this note, we study the effect of external static magnetic field on the formation of the three-dimensional wake potential in a uniformly magnetized dusty plasma with streaming ions. Continuous ion flows are spontaneously established in the sheath plasma where the dust crystals grow in the laboratory experiments. Our intention here is to study, in particular, how the external magnetic field affects the strength of the three-dimensional wake potential and its effective lengths parallel and perpendicular to the direction of ion flow.

We consider the externally applied uniform magnetic field perpendicular to the sheath plane in case (i) and parallel to the sheath plane in case (ii), separately. These are two standard geometries with respect to the directions of the external magnetic field and streaming ions :

(i) External magnetic field along the streaming ions ($z \parallel u_{i0} \parallel B_0$) :

We consider a homogeneous dusty plasma embedded in a uniform external magnetic field perpendicular to the sheath plane where ions are flowing with a constant velocity u_{i0} along the magnetic field lines ($z \parallel u_{i0} \parallel B_0$). This geometry is analogous to that of case *A* of Paper I. The non-Coulombian potential involving the collective interactions between the ion-cyclotron wave and static dust particulate with $k_\parallel v_t \cong 0$ is given by eq. (6) of Paper I.

Now, for the distance greater than the wavelength of the wave involved, we can have $K_\perp \rho/\lambda_D >> 1$. Thus, we can expand $J_0(K_\perp \rho/\lambda_D)$ for large argument, $J_0(K_\perp \rho/\lambda_D) \cong [2/\pi(K_\perp/\lambda_D)\rho]^{1/2} \times \cos(K_\perp \rho/\lambda_D - \pi/4)$. Carrying out the $k_\parallel$-integration, we obtain

$$\phi_w(\rho, z) = \frac{2q_t}{M\lambda_D}\sqrt{\frac{2\lambda_D}{\pi\rho}}\Bigg] I_\perp \tag{1}$$

where

$$I_\perp = \int_0^1 \frac{K_\perp^{5/2}}{\left(K_\perp^2 + 1/f\right)^{1/2}} \cdot \sin\frac{\left[\left(K_\perp^2 + 1/f\right)^{1/2}}{M\lambda_D}$$

$$\cos\left\{\frac{K_\perp \rho}{\lambda_D} - \frac{\pi}{4}\right\} dk_\perp . \tag{2}$$

Now, we consider two important limits for a sufficiently long wavelength waves ($K_\perp << 1$) in the high-density plasma ($\omega_{pi}^2/\omega_{ci}^2 >> 1$).

Firstly for, $K_\perp << 1$ and $K_\perp^2 << 1/f$, we obtain

$$\phi_w(\rho, z) \cong \left[\frac{-2\sqrt{2}\,q_t}{M\sqrt{\pi f}\,\rho}\sqrt{\frac{\lambda_D}{\rho}}\right] \sin\left(\frac{z}{L_\parallel}\right)$$

$$\sin\left(\frac{\rho}{L_\perp} - \frac{\pi}{4}\right) + \frac{1}{\sqrt{2}}\Bigg] \tag{3}$$

where $L_\parallel = \lambda_D M\sqrt{f}$ and $L_\perp = \lambda_D$.

So, for magnetic field parallel to the ion-flow direction for a given low-density plasma, $f \equiv \omega_{pi}^2/\omega_{ci}^2$ decreases with increasing magnetic field and consequently, the amplitude of the wake potential becomes stronger and $L_\parallel$ becomes smaller while $L_\perp$ is unaffected with increasing magnetic field.

Secondly, $K_\perp << 1$ but $K_\perp^2 >> 1/f$ for high-density plasma where $f >> 1$, we obtain

$$\phi_w(\rho, z) \cong \frac{-2q_t}{M\lambda_D}\left(\frac{2}{\pi\rho/\lambda_D}\right)^{1/2} \tag{4}$$

where

$$I_\perp = \cdot\cdot\frac{1}{2}\left(\frac{\cos[x_+/\lambda_D - \pi/4]}{x_+/\lambda_D}\right.$$

$$\left.\cdot\frac{\sin[x_-/\lambda_D + \pi/4]}{x_-/\lambda_D}\right) \tag{5}$$

where $x_\pm = \rho \pm z/M$.

Therefore, for a high-density plasma and relatively smaller magnetic field, $f >> 1$, and consequently, the wake potential becomes independent of the magnetic field with $L_\parallel$ and L⊥ remaining unaffected.

(ii) External magnetic field transverse to streaming ions $(z\|u_{i0} \perp B_0)$ *:*

We consider the case where the external magnetic field is in the x-direction with $k = k_x x + k_y y + k_\parallel z$, where $k_\parallel = k_z z \parallel u_{i0}$ and $k_\perp = \sqrt{k_x^2 + k_y^2}$. Here, the symbol $\parallel$ $(\perp)$ denotes a quantity parallel (perpendicular) to the ion flow which is perpendicular to the external magnetic field.

Now on carrying out the $K_\parallel$-integration, we obtain

$$\phi_w(\rho, z) = -\frac{2q_t}{\lambda_D M^2}\left(\frac{1}{f(M^2-1)}\right)^{1/2}$$

$$\sin\left(\frac{2}{\lambda_D\sqrt{f(M^2-1)}}\right)$$

$$\int J_0\left(\frac{K_\perp \rho}{\lambda_D}\right)K_\perp dK_\perp\,. \tag{6}$$

For $\rho >> \lambda_D$ and expanding J_0, and carrying out the $K\perp$-integration, we obtain

$$\phi_w(\rho, z) \cong \left[\frac{-2q_t}{M^2\sqrt{f(M^2-1)}}\sqrt{\frac{2\lambda_D}{\pi\rho}\frac{1}{\rho}}\right].$$

$$\sin\left(\frac{z}{L_\parallel}\right).\sin\left(\frac{\rho}{L_\perp} - \frac{\pi}{4}\right), \tag{7}$$

where in this case $L_\parallel = \lambda_D\sqrt{f(M^2-1)}$ and $L_\perp = \lambda_D$.

We note from eq. (7) that the ratio of the effective lengths along the ion-flow direction perpendicular to the external magnetic field, between the magnetized and unmagnetized plasmas [15], is given by $L_\parallel / L_s = \sqrt{f} >> 1$ for a high-density plasma. Also, the amplitude of the wake potential gets much reduced from the unmagnetized case for $\omega_{pi}^2/\omega_{ci}^2$. This means that the wake potential with ion-cyclotron waves becomes weak and the effective length for magnetized plasma is large under the laboratory plasma condition because $f >> 1$. The reason why the wake potential becomes weak for magnetized plasma can be explained as follows. The mechanism of attractive forces due to wake potential is quite similar to that of the Cooper pairing. The physical mechanism is the over shielding caused by streaming ions in the downstream direction for the test static negatively charged dust particulate. For magnetized plasmas considered here, the ion motion is more or less influenced by the external magnetic field. Thus, the condition for the over-shielding due to ions, which is necessary for the appearance of the attractive forces, is not easily satisfied. However, for a low-density plasma $(\omega_{pi}^2 << \omega_{ci}^2)$ and magnetic field perpendicular to the ion-flow direction, ϕ_w becomes stronger with $L_\parallel$ smaller and $L_\perp$ unaffected.

In summary, we have investigated wake potential by low-frequency ion-cyclotron wave in a dusty plasma with streaming ions in two standard geometries for the external magnetic field. For a low-density plasma and for a magnetic field parallel to the ion-flow direction (which is perpendicular to the sheath plane), the amplitude of the wake potential increases with smaller $L_\parallel$ for increasing magnetic field. For the high-density plasma $(\omega_{pi}^2 >> \omega_{ci}^2)$ and for magnetic field parallel to the ion-flow direction, the wake potential is nearly unaffected from the unmagnetized case (*cf.* eq. (4)). Again, for the high-density plasma and for external magnetic field perpendicular to the ion-flow direction, the attractive wake potential becomes much smaller than that for the unmagnetized case. The effective length in this case becomes much larger for $\omega_{pi}^2 >> \omega_{ci}^2$. However, the effective length of the wake potential in the direction transverse to the ion-flow direction is the same and is equal to the electron Debye length irrespective of application of the external magnetic field. The predictions of the present note should be verified in the future dust-crystal experiments using external magnetic field.

In the present note, we have studied the formation of the three-dimensional wake potential in the presence of an external uniform magnetic field and a continuous flow of ions in the dusty plasma. The effects of the external static magnetic field on the strength and the effective lengths of the oscillatory wake potential are clearly demonstrated. For magnetic field along the ion flow direction and for the low-density plasma, the lattice spacing in this direction is affected significantly while the spacing in the perpendicular plane is unaffected. The wake field is unaffected for high-density plasma. However, for transverse magnetic field, the wake potential becomes weaker with large (constant) spacing along (perpendicular) the direction of the flow of ions for the high-density plasma. But, for the low-density plasma ($\omega_{pi}^2 << \omega_{ci}^2$), the wake field becomes stronger with small (constant) effective length along the ion-flow direction (perpendicular). Thus the wake potential studied here, can explain the three-dimensional dust-Coulomb crystal formation in the suitable laboratory conditions.

It may be mentioned here that the other effects like $\boldsymbol{E} \times \boldsymbol{B}_0$ drift, the in homogeneity in the plasma, gravitation and collisions, *etc.* on the wake potential, should be important, and the work in this line is in progress.

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